Lecture 5: Channels, Power Spectrum, and AM Modulation

John M Pauly

October 3, 2021

Channels, Power Spectrum, and AM Modulation

Today:

- Channel Equlization
- Autocorrelation and Power Spectral Density
- Amplitude Modulation
- Quadrature Receivers

Next Time:

- Modulators
- Commercial AM
- Single Sideband Modulation (SSB)

Based on Notes from John Gill

Communication Channel Distortion

The linear description of a channel is its impulse response $h(t)$ or equivalently its transfer function $H(f)$.

$$y(t) = h(t) * x(t) \quad \Leftrightarrow \quad Y(f) = H(f)X(f)$$

Note that $H(f)$ both attenuates ($|H(f)|$) and phase shifts ($\angle H(f)$) the input signal.

Channels are subject to impairments:

- Nonlinear distortion (e.g., clipping)
- Random noise (independent or signal dependent)
- Interference from other transmitters
- Self interference (reflections or multipath)

Channel Equalization

Linear distortion can be compensated for by *equalization*.

$$H_{\text{eq}}(f) = \frac{1}{H(f)} \Rightarrow \hat{X}(f) = H_{\text{eq}}(f)Y(f) = X(f)$$

The equalization filter accentuates frequencies that are attenuated by the channel.

However, if $y(t)$ includes noise or interference,

$$y(t) = x(t) + z(t)$$

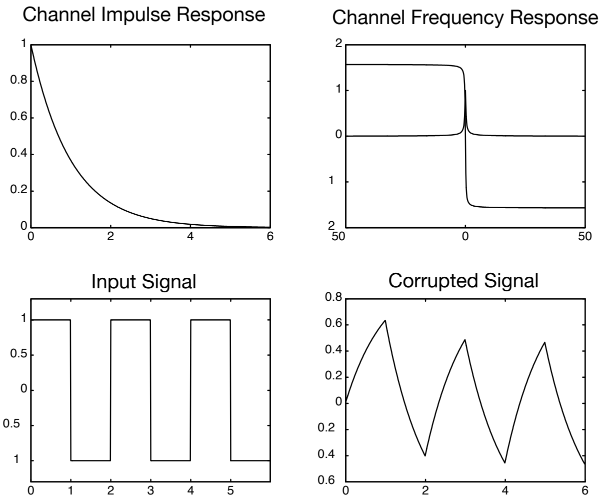
then

$$H_{\text{eq}}(f)Y(f) = X(f) + \frac{Z(f)}{H(f)}$$

Equalization may accentuate noise!

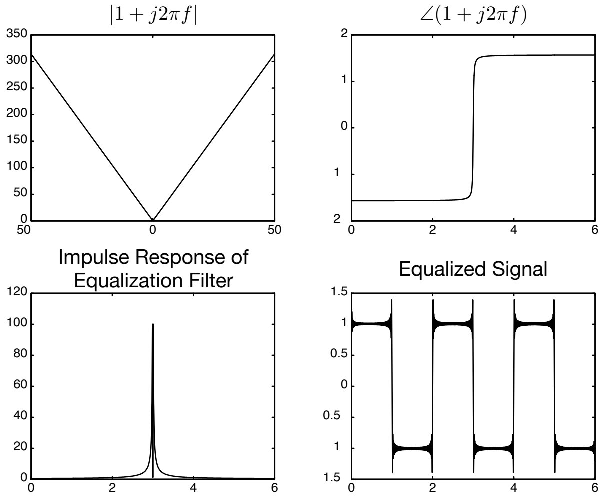
Channel Equalization Example

$h(t) = u(t)e^{-t}$, $x(t)$ is square wave, $y(t) = h(t) * x(t)$.



Channel Equalization Example (cont.)

Equalizing filter has transfer function $1 + j2\pi f$, approximates differentiator.



Signal Energy and Energy Spectral Density

Parseval's theorem for an energy signal $g(t)$ is

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$

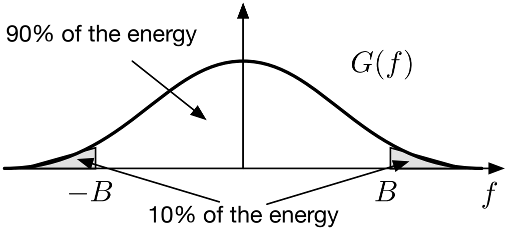
The signal has the same total energy E_g in the time domain or frequency domain.
The *essential bandwidth* as the range of frequencies with most of the energy of the signal. The definition of "most" depends on the application. One choice might be 90%.
If $G(f)$ is a lowpass signal, and E_B is the energy from $-B$ to B, then

$$E_B = \int_{-B}^{B} |G(f)|^2 df$$

Then the essential bandwidth is the B such that

$$E_B/E_g = 0.9$$

This is illustrated below:



Other definitions of width

- 95% or 99% energy
- Half amplitude width
- Half power width
- 50% energy

Autocorrelation and Energy Spectral Density

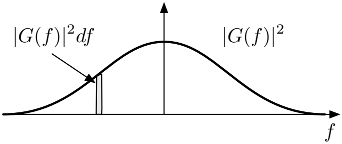
The autocorrelation of a signal $g(t)$ is

$$\psi_g(t) = \int_{-\infty}^{\infty} g(\tau) g^*(t+\tau) d\tau$$

You'll show in your homework that

$$\mathcal{F}\left\{\psi_g(t)\right\} = \left|G(f)\right|^2 = \Psi(f)$$

This is the *energy spectral density* or ESD. It reflects where the energy of the signal is located.

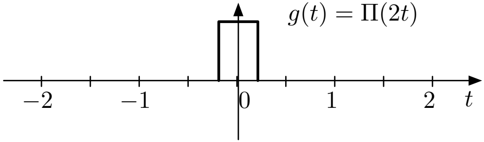


Note that

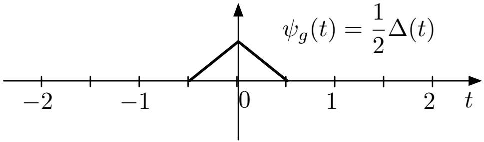
$$E_g = \int_{-\infty}^{\infty} |G(f)|^2 df = \int_{-\infty}^{\infty} \Psi(f) df$$

Energy Spectral Density Example

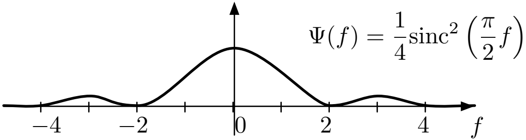
Let $g(t) = \Pi(2t)$



The autocorrelation $\psi(t)$ is



The energy spectral density is then



Autocorrelation and Power Spectral Density

For power signals, we normalize the ESD by the duration, to produce the *power spectral density* or PSD.
The autocorrelation for a power signal $g(t)$ is defined as

$$\mathcal{R}_g(t) = \lim_{T\to\infty} \frac{1}{T} \int_{T/2}^{T/2} g(\tau) g^*(t+\tau) d\tau$$

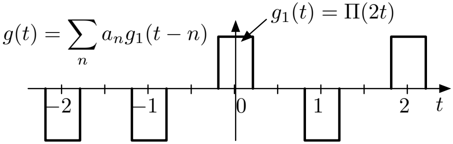
This has the Fourier transform

$$\mathcal{F}\left\{\mathcal{R}_g(t)\right\} = \lim_{T\to\infty} \frac{1}{T} \Psi_{g,T}(f) = \mathcal{S}_g(f)$$

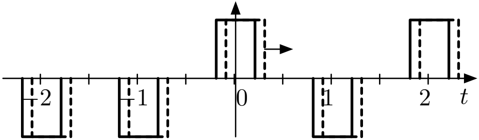
$\mathcal{S}_g(f)$ is the power spectral density, PSD.
Again, this shows where the frequency distribution of the power of the signal.

Power Spectral Density Example

Let $g(t)$ be a random binary sequence of rectangle pulses $g_1(t) = \Pi(2t)$

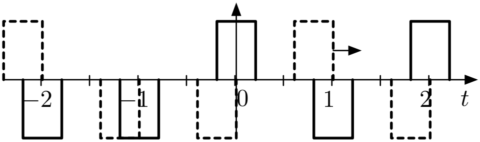


The for small displacements, the autocorrelation looks like

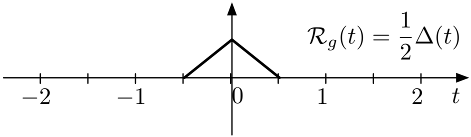


After normalizing by the interval T, this will be the same a for a single pulse.

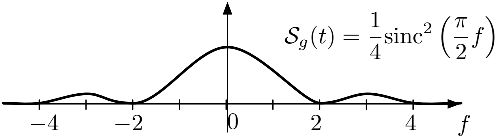
The for large displacements, the overlaps are just as likely to be ± 1, and will cancel.



This will go to zero as T gets large.
The autocorrelation is then



The power spectral density is then



Baseband Communication

The *baseband* is the frequency band of the original signal.

- Telephones: 300–3700 Hz
- High-fidelity audio: 0–20 KHz
- Television (NTSC) video: 0–4.3 MHz
- Ethernet (10 Mbs): 0–20 MHz

Baseband communication usually requires wire (single, twisted pair, coax).

Multiple baseband signals cannot share a channel without time division multiplexing (TDM).

Carrier Communication

Carrier communication uses modulation to shift spectrum of signal.

- Wireless communication requires frequencies higher than baseband
- Multiple signals can be sent at same time using different frequencies: frequency division multiplexing (FDM)

In carrier communication, the signal modulates a sinusoidal carrier.
The signal modifies the amplitude, frequency, or phase of carrier.

$$s(t) = A(t)\cos\big(2\pi f_c(t)t + \phi(t)\big)$$

- amplitude modulation: $A(t)$ is proportional to $m(t)$
- frequency modulation: $f_c(t)$ is proportional to $m(t)$
- phase modulation: $\phi(t)$ is proportional to $m(t)$

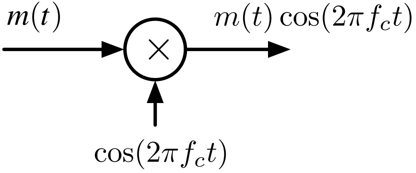
Frequency and phase modulation are called *angle modulation*.

Double-Sideband Suppressed Carrier (DSB-SC) Modulation

- This is a complicated way of saying "multiply by a cosine."
- We have a message (baseband) signal, and cosine $\cos(2\pi f_c t)$ at a frequency f_c.
- The modulated message signal and its Fourier transform are

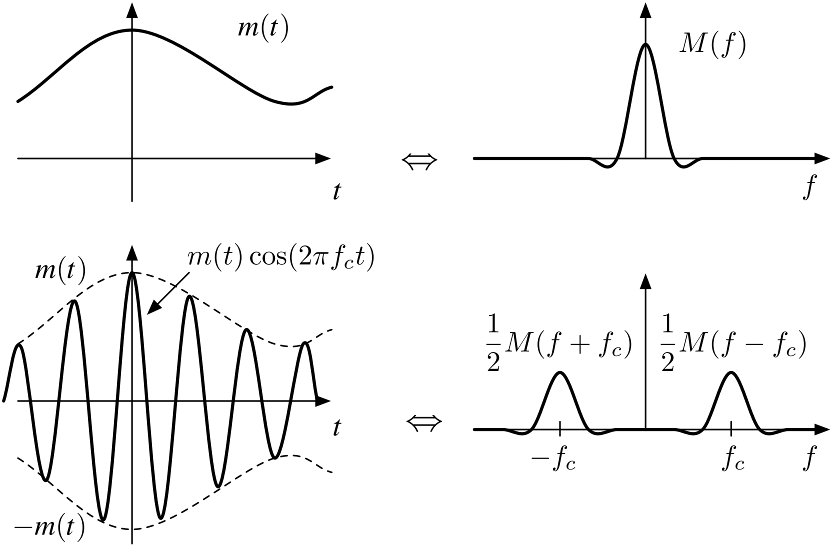
$$m(t)\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}\left[M(f+f_c)+M(f-f_c)\right]$$

where $m(t) \Leftrightarrow M(f)$. The block diagram is



- This signal and its spectrum are illustrated on the next page:

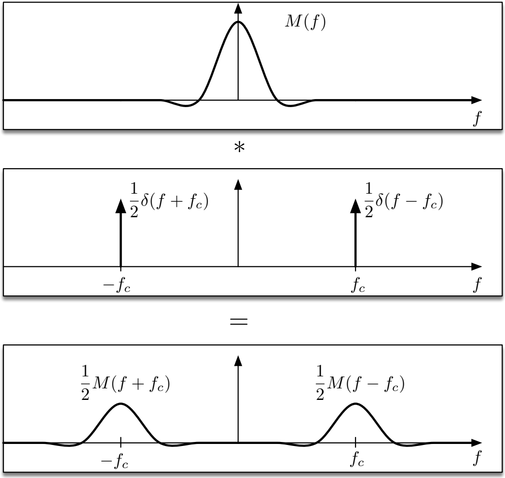
DSB-SC



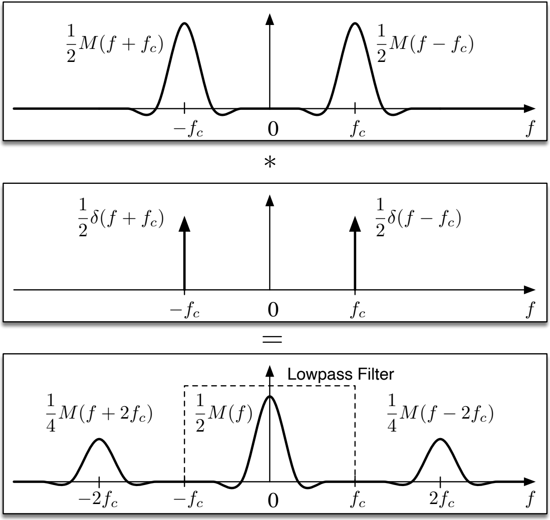
$m(t)$
t
$\Leftrightarrow$
$M(f)$
f
$m(t)$
$m(t)\cos(2\pi f_c t)$
$-m(t)$
t
$\Leftrightarrow$
$\frac{1}{2}M(f+f_c)$
$\frac{1}{2}M(f-f_c)$
$-f_c$
f_c
f

We can think of modulation as frequency domain convolution.

$$\mathcal{F}\left[m(t)\cos(2\pi f_c t)\right] = M(f) * \left(\frac{1}{2}\delta(f+f_c) + \frac{1}{2}\delta(f-f_c)\right)$$



To demodulate this signal, consider what happens if we multiply again by $\cos(2\pi f_c t)$. Again, we can think of this as a convolution in the frequency domain:



After the convolution there is a replica of the spectrum centered at $f = 0$, which we can extract with a lowpass filter.
The modulated signal spectrum is

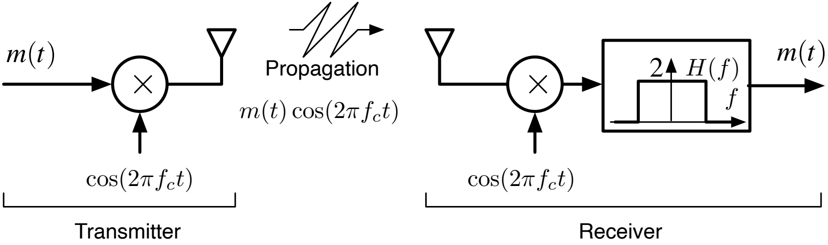
$$\mathcal{F}\left[m(t)\cos(2\pi f_c t)\right] = \frac{1}{2}M(f+f_c) + \frac{1}{2}M(f-f_c)$$

Multiplying this by $\cos(2\pi f_c t)$ corresponds to convolving in frequency,

$$\begin{aligned}
\mathcal{F}\left[m(t)\cos^2(2\pi f_c t)\right] &= \left[\frac{1}{2}M(f+f_c)) + \frac{1}{2}M(f-f_c)\right] \\
&\quad * \left[\frac{1}{2}\delta(f+f_c) + \frac{1}{2}\delta(f-f_c)\right] \\
&= \frac{1}{4}\left[M(f+f_c) + M(f-f_c))\right] \\
&\quad * \left[\delta(f+f_c) + \delta(f-f_c)\right] \\
&= \frac{1}{4}M(f+2f_c) + \frac{1}{2}M(f) + \frac{1}{4}M(f-2f_c)
\end{aligned}$$

Lowpass filtering extracts the $M(f$ term, recovering the original message.

The block diagram of the entire system is now:



- This is a *synchronous* receiver, meaning the transmitter and receiver must be in phase. Synchronising the receiver requires a more complex system.
- In practice the propagation delay is unknown and time varying, and the transmitter and receiver phase can drift with respect to each other.

DSB-SC Synchronization and Quadrature Receivers

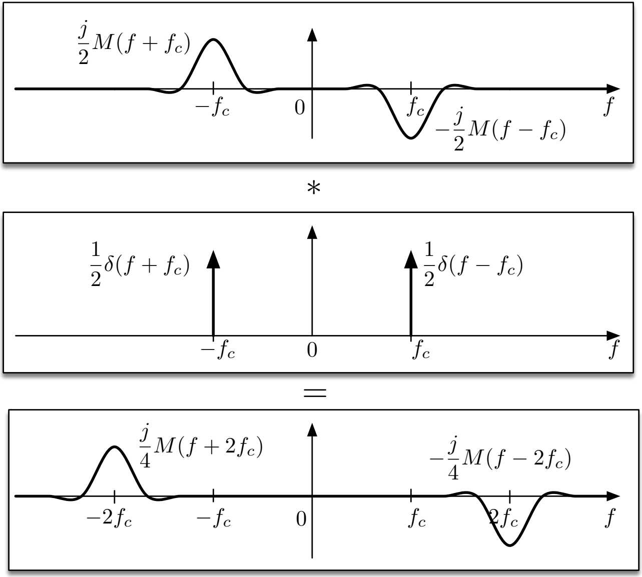
Often we won't know the phase of the signal we are receiving, and this can cause problems with a synchronous receiver.
Consider the case where the transmitter has a phase of $-\pi/2$, so that the modulated signal is

$$m(t)\cos(2\pi f_c t - \pi/2) = m(t)\sin(2\pi f_c t).$$

The spectrum of the transmitted signal is now

$$\mathcal{F}\left[m(t)\sin(2\pi f_c t)\right] = \frac{j}{2}M(f+f_c) - \frac{j}{2}M(f-f_c)).$$

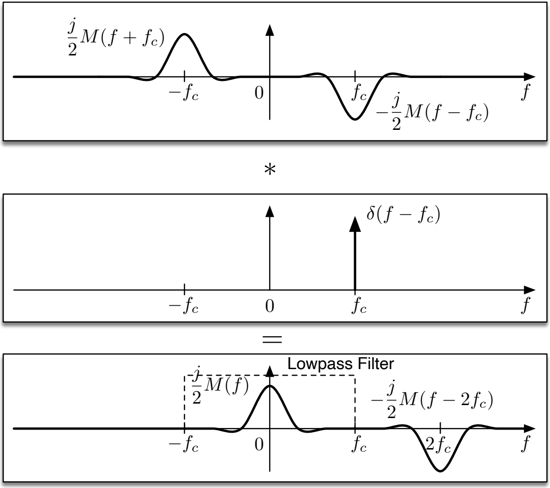
If we demodulate with a cosine, the result is shown in the next plot:



The baseband signal we want cancels!

One solution is to demodulate with a complex exponential

$$e^{j2\pi f_c t} \Leftrightarrow \delta(f - f_c)$$



This also works for an arbitrary phase shift.

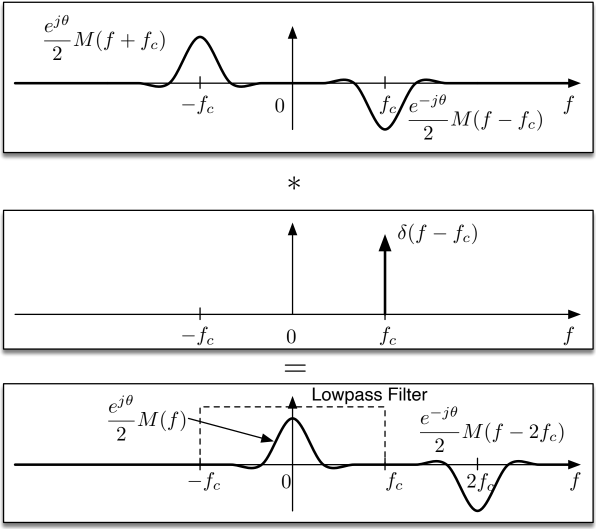
If the input has a phase shift of $-\theta$, the spectrum of the carrier is

$$\begin{aligned}
\mathcal{F}\left[\cos(2\pi f_c t - \theta)\right] &= \mathcal{F}\left[\cos(2\pi f_c t)\cos\theta + \sin(2\pi f_c t)\sin\theta\right] \\
&= \frac{1}{2}\cos\theta\left[\delta(f+f_c) + \delta(f-f_c)\right] \\
&\quad + \frac{1}{2}\sin\theta\left[j\delta(f+f_c) - j\delta(f-f_c)\right] \\
&= \frac{1}{2}\left[e^{j\theta}\delta(f+f_c) + e^{-j\theta}\delta(f-f_c)\right]
\end{aligned}$$

and the spectrum of the modulated signal is

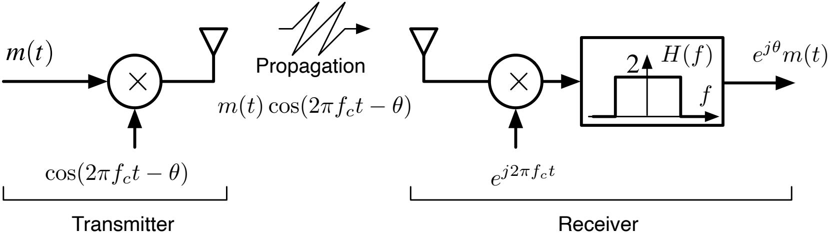
$$\mathcal{F}\left[m(t)\cos(2\pi f_c t - \theta)\right] = \frac{e^{j\theta}}{2}M(f+f_c) + \frac{e^{-j\theta}}{2}M(f-f_c)$$

Demodulating with a complex exponential can be plotted as:



The lowpass filter extracts $\frac{e^{j\theta}}{2}M(j\omega)$ corresponding to the complex signal $\frac{e^{j\theta}}{2}m(t)$.

The system now looks like



- This is a *quadrature receiver*, common in radar, sonar, ultrasound, and MRI systems. Often $m(t)$ is a simple pulse, and the interesting information is in θ, such as doppler shift for weather radar.
- This also common in communications systems. You can buy a digital chip that implements a quadrature receiver for your cell phone.
- The cost is that the receiver has to be implemented for complex signals. This is done the way you do it, by keeping track of two real signals, the real part and the imaginary part (the I and Q channels).

New Time

- Modulators, and how to make them
- Commercial AM
- Single Sideband Modulation (SSB)