Lecture 8: Angle Modulation

John M Pauly

October 12, 2021

Modulation

- Modulation encodes a real message $m(t)$ on a carrier $\cos(2\pi f_c t)$
- There are many ways to do this.
- So far we've looked at various amplitude modulation methods such as AM, SSB, or QAM. Here a carrier is multiplied by a *real* envelope.
- We can also encode information in the phase or frequency of the carrier.
- This can be described as a carrier multiplied by a *complex* envelope.

Based on lecture notes from John Gill

Amplitude Modulation

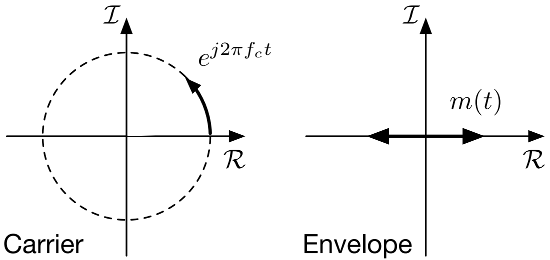
The DSB-SC modulated signal is

$$2m(t)\cos(2\pi f_c t) = m(t)e^{j2\pi f_c t} + m(t)e^{-j2\pi f_c t}$$

If we just focus on the positive frequency term we have

- The envelope which is the message $m(t)$ multiplied by
- The carrier $e^{j2\pi f_c t}$

If we plot these in the complex plane at some time t, this looks like



The message $m(t)$ scales the amplitude of the carrier.

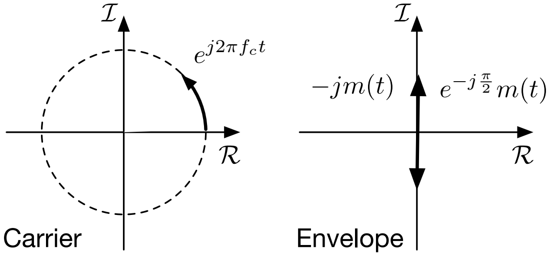
If we modulate a sine instead,

$$2m(t)\sin(2\pi f_c t) = -jm(t)e^{j2\pi f_c t} + jm(t)e^{-j2\pi f_c t}$$

Again we just focus on the positive frequency term, which we can consider to be

- The envelope $-jm(t)$ which includes the message $m(t)$
- The same carrier $e^{j2\pi f_c t}$

Note that the fact that we multiplied by a sine is reflected in the $-j$ in the *envelope*. The carrier is the same. This looks like



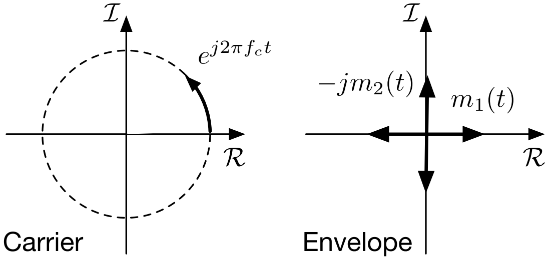
The envelope is now imaginary (the $-j$) or can be thought of as phase shifted by $-\pi/2$. This turns the cosine carrier into a sine.

With QAM we have two messages, one modulated by a cosine, and a second with a sine

$$
\begin{aligned}
2m_1(t)\cos(2\pi f_c t) \quad &+ \quad 2m_2 \sin(2\pi f_c t) \\
&= \quad (m_1 - jm_2(t))e^{j2\pi f_c t} + (m_1(t) + jm_2(t))e^{-j2\pi f_c t}
\end{aligned}
$$

The positive frequency term is then

- The complex envelope $m_1(t) - jm_2(t)$ which has $m_1(t)$ as the real part, and $m_2(t)$ as the imaginary part.
- The same carrier $e^{j2\pi f_c t}$.

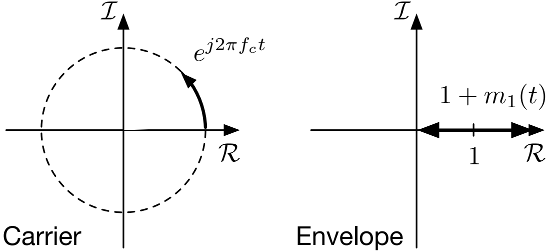


The magnitude of $m_1(t) - jm_2(t)$ scales the amplitude of the carrier, and the angle adds to the phase of the carrier.

Broadcast AM is like DSB-SC with an additional bias which makes the envelope always positive

$$2(1+m(t))\cos(2\pi f_c t) = (1+m(t))e^{j2\pi f_c t} + (1+m(t))e^{-j2\pi f_c t}$$

This looks like



We are transmitting the AM bias term all the time.
Is there something else we could do with it?

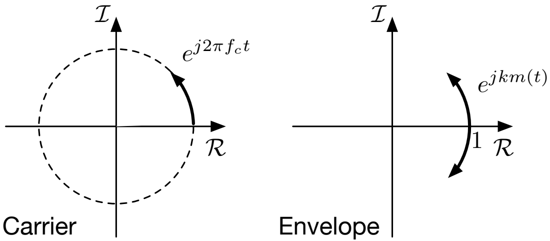
Instead of mutiplying the carrier by the message, we can include the message as the phase of the carrier

$$
\begin{aligned}
2\cos(2\pi f_c t + km(t)) &= e^{j(2\pi f_c t + km(t))} + e^{j(2\pi f_c t + km(t))} \\
&= e^{jkm(t)} e^{j2\pi f_c t} + e^{-jkm(t)} e^{j2\pi f_c t}
\end{aligned}
$$

Now we have

- A complex envelope $e^{jkm(t)}$, which has unit magnitude and phase $km(t)$
- The same carrier $e^{j2\pi f_c t}$.

This looks like



For small angles, this looks like the quadrature version of AM!
Could we do something like QAM here?

Angle Modulation

- We can encode information as either
  - Time varying phase
  - Time varying frequency
- Both result in angle modulation
- Both are very closely related
- Easy to do with an sdr, just another complex envelope!

Instantaneous Frequency

- In general, the frequency of a signal at an instant in time depends on the entire signal.
- For generalized sinusoids, we can use a simpler approach. Suppose

  $$\varphi(t) = A\cos\theta(t)\,.$$

  Then $\theta(t)$ is the *generalized angle*. For a true sinusoid,

  $$\theta(t) = \omega_c t + \theta_0\,,$$

  linear with slope ω_c and offset θ_0
- The generalized angle is *not* limited to $[0, 2\pi]$. Wrapping introduces discontinuities.
- Phase unwrapping is easy at an IF, where the phase changes are small from on sample to the next.

Instantaneous Frequency (cont.)

- Instananeous frequency is derivative of generalized angle:

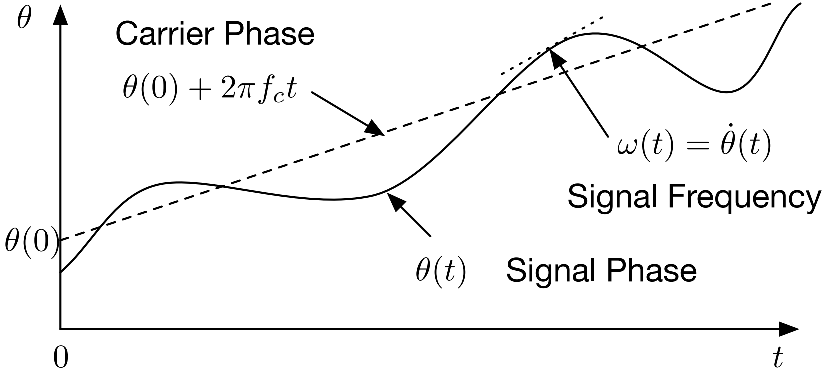
$$\omega_i(t) = \frac{d\theta}{dt} = \theta'(t)$$

- The phase is just the integral of the frequency

$$\theta(t) = \int_{-\infty}^{t} \omega_i(u)\, du = \theta(0) + \int_{0}^{t} \omega_i(u)\, du$$

- We can modulate a generalized sinusoid by using a signal $m(t)$ to vary either $\theta(t)$ or $\omega_i(t)$.
- In either case, the frequency of the modulated signal changes as a function of $m(t)$.

Instantaneous Frequency (cont.)



- Dashed line is the carrier phase
- Solid line is the phase of the transmitted signal
- Slope of the solid line is the instantaneous frequency of the transmitted signal.

Phase Modulation (PM)

- In PM, *phase* is varied *linearly* with $m(t)$:

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

  which produces a transmitted signal

$$\varphi_{\mathrm{PM}}(t) = \cos\big(2\pi f_c t + k_p m(t)\big)$$

- The instantaneous frequency is

$$\omega_i(t) = \frac{d\theta}{dt} = 2\pi f_c + k_p \dot{m}(t)$$

- If $m(t)$ varies rapidly, then the frequency deviations are larger.
- The bandwidth of the signal is determined by $\dot{m}(t)$, similar to AM.

Frequency Modulation (FM)

- In FM, *frequency* is is varied *linear* in $m(t)$:

$$\omega_i(t) = 2\pi f_c + k_f m(t)$$

  which produces a signal

$$\varphi_{\mathrm{FM}}(t) = \cos\big((2\pi f_c + k_f m(t))t\big)$$

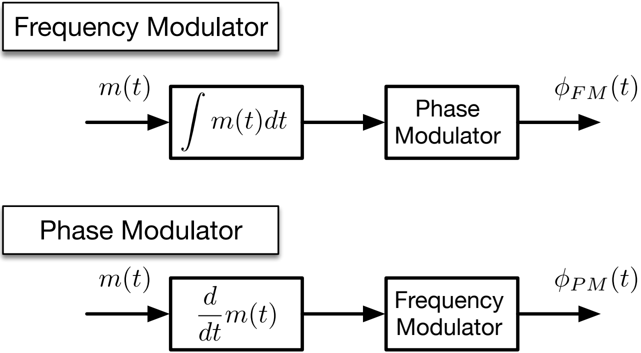
- The bandwidth of the signal is determined by the *amplitude* of $k_f m(t)$ (obvious), and also the bandwidth of $m(t)$ (less obvious).
- The angle is

$$\theta(t) = \int_{-\infty}^{t} \big(2\pi f_c + k_f m(t)\big)\, du = 2\pi f_c t + k_f \int_{-\infty}^{t} m(u)\, du$$

- We could apply this $\theta(t)$ to a phase modulator, and get exactly the same effect as applying $\omega_i(t)$ to a frequency modulator.

Relationship Between FM and PM

- Phase modulation of $m(t)$ = frequency modulation of $\dot{m}(t)$.
- Frequency modulation of $m(t)$ = phase modulation of $\int m(u)\,du$.



- We can produce both types of modulation with either modulator.
- Direct digital synthesis (DDS) chips will do this for you.

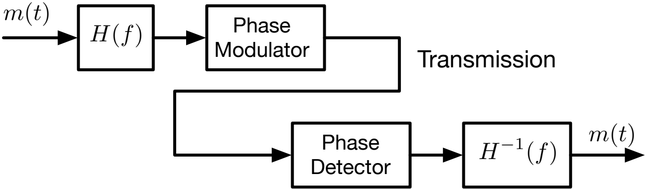
Generalized Angle Modulation

- We can generalize modulation by convolving the message signal with an impulse response $h(t)$

$$\varphi_{\text{EM}}(t) = A\cos\left(2\pi f_c t + h(t) * m(t)\right)$$

  This is a filter with a transfer function $H(f)$.

- We recover $m(t)$ from phase by using inverse filter $H^{-1}(f)$. E.g., for FM the inverse of integration is differention.

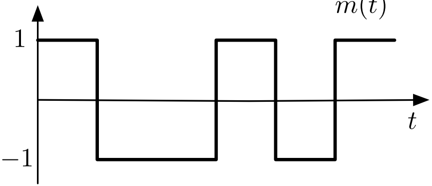


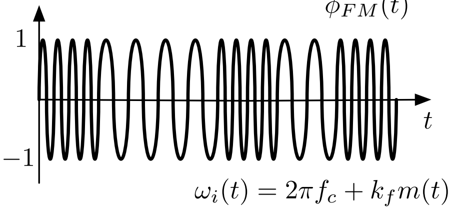
- PM $(h(t) = k_p\delta(t))$ and FM $(h(t) = k_f u(t))$ are special cases.

- Also used for pre-emphasis to improve noise characteristics, and pulse shaping to reduce signal bandwidth.

Frequency Shift Keying (FSK)

- Basic idea is to send a string of bits as two different frequencies
- These are encoded in $m(t)$ as a value of 1 for a bit 1, and a value of -1 for a bit 0.
- We then transmit a frequency $2\pi f_c t + k_f$ for a 1, and $2\pi f_c - k_f$ for a zero.

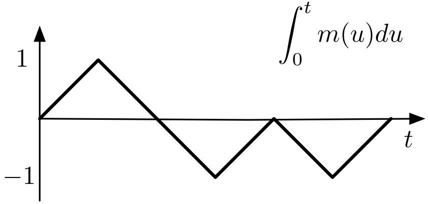


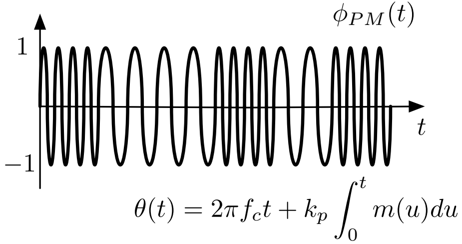


- This type of modulation is very common in modems, and also digital radio
- We'll see this later in one of the labs.

FSK with a Phase Modulator

- We could achieve the same effect with a phase modulator.
- The input is now $\int_o^t m(u)du$

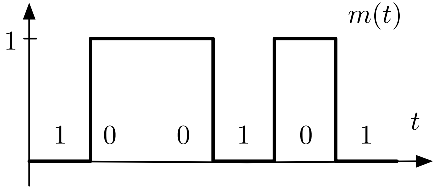


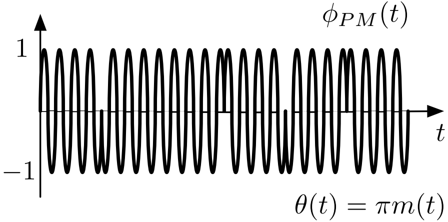


- This results in exactly the same waveform given k_p is properly scaled.

Phase Shift Keying (PSK)

- Basic idea is to send a string of bits as two (or more) phases of a carrier
- These are encoded in $m(t)$ as a value of 0 for a bit 1, and a value of 1 for a bit 0, which is then scaled to 0 and π.
- This inverts the carrier for zero bits

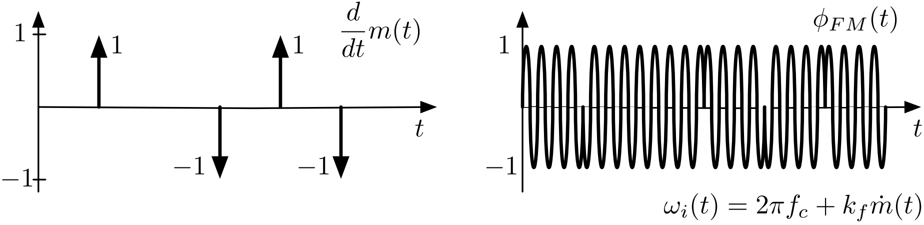




- This is also common for digital radio and modems
- Some car key fobs used this. We will also see this later.

PSK with a Frequency Modulator

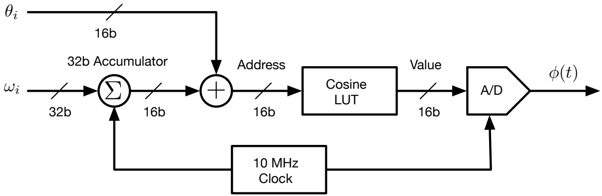
- We could achieve the same effect with a frequency modulator
- The input is now $\frac{d}{dt}m(t)$.



- Again, this results in exactly the same waveform given k_f is properly scaled.

Direct Digital Synthesis

Direct Digital Synthesis (DDS) system block diagram



- LUT is 2^{16} samples of a cosine. It takes an index and outputs the value
- Generating the FM or PM waveform consists of stepping through this array at the right speed
- The 10 MHz clock sets the speed. Every 0.1 μ s there is a new output
- The step size is set by ω_i. For the current $\omega(t)$, how much does the address advance in 0.1 μ
- The phase θ_i is just an offset into the $[0..2^{16}]$ samples for $[0..2\pi]$.

Next Time

- Narrowband FM (Police, Amateur Radio)
- Wideband FM (Commercial FM)
- Spectral bandwidth of FM signals
- Modulation and demodulation systems